New Economic School
Microeconomics 5
Make-Up Examination

This is an A4 exam. You have three hours to answer **ANY THREE** of the four questions below. All three questions you answer have equal weight. Good luck!

1. There is one worker on the labor market and a few competitive firms ready to hire him. Worker's productivity θ is known to the worker but not to firms and can assume value $\theta_H = 3$ with prior probability λ and value $\theta_L = 1$ with prior probability $1 - \lambda$. If unemployed, worker of type θ_L may earn $r(\theta_L) = 0$ and worker of type θ_H may earn $r(\theta_H) = r \geq 0$ at home.

   A worker may acquire some education e prior to going on the market; this education is irrelevant to his future productivity. Acquiring education level e for the worker of type θ costs $\frac{e}{\theta}$.

   (a) Find all separating and pooling equilibria and derive restrictions on r and λ under which they exist.

   (b) For each equilibrium you found in (a) show whether a social planner (who does not know worker's type θ) can intervene and (weakly) Pareto improve upon it (your answer would depend on r and λ).

2. There is a principal and an agent. The agent may choose either to work (exert effort $e = 1$) or not (effort $e = 0$); exerting no effort $e = 0$ costs c_0 to the agent while exerting effort $e = 1$ costs c_1 to the agent (in utility terms). Agent's effort is not observable and his reservation utility is. Principal's (verifiable) profit π in either case is a random variable distributed with Rayleigh CDF $F(\pi|e) = 1 - \exp\left(-\frac{\pi^2}{2\lambda_e^2}\right)$ for $\pi \geq 0$ and positive constants λ_0 and λ_1. The principal makes a take-it-or-leave-it offer in the form $w(\pi) \geq 0$ to the agent. The principal is risk neutral and the agent's utility of wage is $v(w) = \frac{2\sqrt[3]{w^2}}{3}$.

   (a) What is the condition on λ_0 and λ_1 that guarantees $F(\pi|1)$ FOSD $F(\pi|0)$? **For the rest of the question assume that this condition holds.**

   (b) What are the optimal full information ("first best") contracts for high and low desired level of effort? What is the first best level of effort, depending on $c_1 - c_0$?

   (c) Solve for the optimal $w(\pi)$ for high level of effort under asymmetric information, assuming that both IC and IR conditions are binding for high level of effort. What is the condition on c_1 and c_0 under which both IC and IR will indeed be binding?

   Hint: For Rayleigh distribution with cdf $F(\pi|e) = 1 - \exp\left(-\frac{\pi^2}{2\sigma^2}\right)$ the mean is $\sigma\sqrt{\frac{\pi}{2}}$ and the variance is $\frac{4-\pi}{2}\sigma^2$.

3. The government P regulates a (privately owned) natural monopoly A. The monopoly's cost of production depends on a technology parameter c is its private information and can take three values $c = 1$, $c = 2$, and $c = 3$, with equal probability. P's utility of consuming q units of A's output and paying the total of p dollars is $U = q - p$. A's payoff is $p - cq^2/2$.

   (a) Find the optimal contract (or menu of contracts) p, q that P offers A under asymmetric information.

   (b) What would the optimal contract be if P observed A's type?

4. Consider the following average-price sealed-bid auction for one object. There are two bidders. Each bidder i has valuation θ_i, which is uniformly distributed on $[0, 1]$. All valuations are independent and they are unobservable by the other players. The rules are the following. Each bidder i submits her bid b_i. The object goes to the bidder with the highest bid; the winner has to pay $(b_1 + b_2)/2$.

   (a) Find the symmetric Nash Bayesian equilibria in linear strategies.

   (b) Compare to the second-price sealed bid auction and check whether revenue eqivalence holds.