MICROECONOMICS-4 (2005): MAKE-UP EXAM

You have four problems and three hours to solve them. Using any books or notes is not permitted. Good luck!

1. (20 points). Consider an economy with 2 consumers, 1 firm and 3 goods. Consumers' utility functions are $u_1(x_{11}, x_{21}, x_{31}) = \ln x_{11} + \ln x_{21}$ and $u_2(x_{12}, x_{22}, x_{32}) = \ln x_{12} + \ln x_{32}$. Initial endowments are $\omega_1 = (0, 24, 0)$ and $\omega_2 = (0, 0, 24)$. The firm uses goods 2 and 3 to produce good 1; it has CRS technology $y_1 = \frac{1}{5}y_2 + y_3$. Consumer own equal shares in the firm's profits. Find competitive equilibria in this economy.

2. (30 points).

   Consider an exchange economy with I agents, each having rational preferences $\succeq_i$on $\mathbf{R}^L_+$. The total endowment of goods in the economy is $\bar{\omega} = (\bar{\omega}_1, ...\bar{\omega}_L) \gg 0$, each agent's endowment is $\omega_i \in \mathbf{R}^L_+$. We shall say that agent i *envies* agent j in allocation $(x_1, ...x_I)$ if $x_j \succ_i x_i$, that is i prefers j's bundle. A feasible allocation $x = (x_1, ..., x_I)$ is called *envy-free*, if there is no pair of agents i, j such that i envies j. We shall call a feasible allocation *fair*if it is Pareto-efficient and envy-free.

   (a) (10 points) Suppose that preferences are continuous and strongly monotone. Show that if (x, p) is a competitive equilibrium such that $px_i = px_j$ for all i, j, then x is fair.

   (b) (10 points) Show that if preferences are continuous, convex and strongly monotone, fair allocations exist.

   (c) (10 points) Prove that if a feasible allocation $(x_1, ..., x_I)$ is Pareto-efficient, then there is some agent that envies no-one and there is some agent that no one envies.

3. (20 points) Consider an economy with two consumers 1,2 and three goods x, y, z. Consumer 1 initially has one unit of good x and consumer 2 has one unit of good y. Each consumer's utility over consumption bundles (x, y, z) is $x^{2/9}y^{2/9}z^{5/9}$. Each consumer has access to the following CRS technology: he is able to produce a unit of good z from a unit of good x and a unit of good y (this is a special case of Leontieff technology; the corresponding production function is $z = \min(x, y)$).

   (a) (7 points) Find the core of this economy.

   (b) (13 points) Find the core of the double replica of this economy.

4. (30 points) There is a model of exchange economy under uncertainty with three states of nature A, B, C and three dates $t = 0, 1, 2$. At date $t = 0$, the state of nature is unknown. At date $t = 1$, some of uncertainty reveals: everyone gets information on whether state A will occur, but so far nobody can make difference between states B and C. At date $t = 2$, the state of nature is known.

   At date $t = 0$, the financial market opens. Assets traded in this market are titles to receive some amount of numeraire good (dollars) in each state at date $t = 2$ (before opening the spot market). The asset structure is the following:

   Asset 1: "riskless" asset, returns \$1 in each state.

   Asset 2: returns \$6 in state A, \$5 in state B, \$1 in state C.

   Asset 3: gives a right to buy a unit of asset 2 at price 2 at date $t = 2$ (after all uncertainty reveals but before the spot trade).

   Asset 4: gives a right to buy a unit of asset 2 at price 2 at date $t = 1$.

   It is known that the price of asset 1 is \$1 and the price of asset 2 is \$3 at date $t = 0$.

   Find the range of all possible arbitrage-free prices for the following cases:

   (a) (10 points) price of asset 3 at date $t = 0$;

   (b) (10 points) price of asset 4 at date $t = 0$;

   (c) (10 points) price of asset 3 at date $t = 1$.