New Economic School
Microeconomics 3
Problem set #6
Due Week 7

The problems below refer to the material in MWG, Ch.6 and Nicholson, Chapter 25.

1. The agent can invest his portfolio w in two risky assets. There are two states of nature that are equally likely. The first asset brings return brings y_1, y_2 in the states 1,2, respectively, and the second asset brings z_1, z_2. The agent's Bernoulli utility is $u(x) = \ln x$.

   (a) Find the optimal portfolio if the assets' returns are positively correlated: $(y_1 - y_2)(z_1 - z_2) > 0$.

   (b) Find the optimal portfolio if the assets' returns are negatively correlated: $(y_1 - y_2)(z_1 - z_2) < 0$.

2. There are two individuals with logarithmic Bernoulli function $u(x_i) = \ln x_i$, $i = 1, 2$. Each individual has an endowment θ_i, where θ_i are i.i.d.; each θ_i takes values 1 and e with equal probability. They need to agree on a mechanism for financing a public good. The cost of public good is 1; if the public good is not financed, both receive zero income hence $-\infty$ utility. The timing is as follows: (i) the parties agree on a mechanism $x_i(\theta_1, \theta_2)$, (ii) each individual learns her θ_i; (iii) the mechanism is played so that each agent receives $u(x_i(\theta_1, \theta_2))$. Both θ_1, θ_2 are observable at stage (iii). Notice that the mechanism should satisfy $x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) = \theta_1 + \theta_2$ for every realization of θ_1 and θ_2.

   (a) Find the utility possibility frontier at stage (i) (in the space of expected utility). Find the egalitarian arrangement.

   (b) Find the utility possibility frontier and the egalitarian arrangement for each realization θ_1, θ_2. Can (a) be represented as an expectation of (b)?

3. Each agent in an economy is endowed with a unit of labor. Agent may choose either of two activities: production or rent-seeking. If used in production, a unit of labor yields return π. The productivity π is distributed among agents according to c.d.f. $F(\pi)$ ($F(0) = 0, F(\infty) = 1$). The agents who choose to become rent-seekers get $\frac{L_r}{G+L_r}$ share of producers' (after tax) income. Here L_r is total labor committed to rent-seeking and G is a public good (law and order) provided by the government. Each rent-seeker gets an equal share of the rent obtained. The public good provision is financed through a proportional tax on producers' income with tax rate $t \in [0, 1]$. The production function is $G(T) = A\sqrt{T}$, where T is the tax revenue.

(a) Given a tax rate t, characterize the equilibrium. Is it unique? How does occupational choice depend upon the tax rate? Find national income and consumption in equilibrium.

(b) What tax rate minimizes rent-seeking? Compare it to the tax rate that maximizes consumption.

(c) What tax rate would be chosen by an agent with productivity π_m? Find a distribution F for which the median voter would choose a tax rate that maximizes aggregate consumption.