New Economic School
Microeconomics 3
Problem set #3
Due Week 4

The problems below refer to the material in MWG Chapters 3-4.

1. There are two goods in the economy, x and y; the prices are p and q, respectively. There are two consumers $i = 1, 2$ with wealth w_i. Each consumer has a Cobb-Douglas utility function $u_i = \sqrt{x_i y_i}$. The social welfare function is $W = u_1 + u_2 + \sqrt{u_1 u_2}$.

   (a) Derive individual Marshallian (uncompensated) and Hicksian (compensated) demand functionfor individual consumers. Compute Slutsky matrices. Do individual demands satisfy the uncompensated law of demand?

   (b) Derive aggregate demand. Does it satisfy uncompensated demand law? The Weak Axiom? If a (positive) representative consumer exists, describe her utility function.

   (c) Solve the social planner's problem. Describe a (normative) representative consumer.

2. There are two consumers with indirect utility functions $v_1(p, w_1)$ and $v_2(p, w_2)$. Answer the questions below, provide a proof for positive answers and countrerexamples for negative ones.

   (a) Does the aggregate demand exists? If it does, is it generated by the indirect utility function $v(p, w) = v_1(p, w_1) + v_2(p, w_2)$.

   (b) Answer (a) for $v_i = f(p)w_i$, $i = 1, 2$.

3. There are two goods in the economy, x and y; the prices are p and q, respectively. There are two consumers $i = 1, 2$ with wealth w_i. Each consumer has an indirect utility function $v_i = w_i \frac{pq}{p+q}$.

   (a) Does the aggregate demand satisfy the Weak Axiom?

   (b) Compute the aggregate demand. Derive the representative consumer's utility function.

   (c) Find the social welfare function such that the solution to the social planner's maximization problem is the same as (b).