

Macroeconomics IV. 2004-2005. Homework 3.
Due: Monday, April 18, 5:00pm.

1 Optimal investment under uncertainty.

Consider a firm with Cobb-Douglas production technology, $Y_t = AK_t^\alpha L_t^{1-\alpha}$, depreciation rate δ , and quadratic costs of adjustment of capital, $C(I) = bI^2$; the law of motion for capital is:

$$K_t = (1 - \delta)K_{t-1} + I_t.$$

The price of new capital goods in terms of output is equal to one. The firm operates in competitive goods and labor markets and takes the real wage, w_t , as given. Wages are the only source of uncertainty. The firm is owned by risk neutral owners, with subjective discount rate θ .

a) Assuming that the firm chooses employment freely in each period, solve for profit at time t given K_t and w_t .

b) Show how optimal investment depends on current and future expectations of wages.

c) Assume that the wage follows a Markov chain with two states, w_1 and w_2 , and transition matrix:

$$T = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}.$$

Derive the process followed by optimal investment. Characterize the effects of p and q on investment. Explain.

d) Find the steady state distribution of capital.

2 Credit, reputation, and investment.

Consider an economy which consists of a unit continuum of risk-neutral infinitely-lived entrepreneurs and a unit continuum of risk-neutral investors, who live one period only. Entrepreneurs have no funds, which they must

borrow from investors (assume that entrepreneurs get no utility from consumption of the funds they borrow). Given the funds, entrepreneurs can undertake risky projects. Investors can invest either in projects or in a riskless asset, which yields gross return $1 + r$.

Assume that there are two types of projects — good and bad, and two types of entrepreneurs — able and incapable. All projects require a unit investment; bad projects result in a total loss of invested funds, whereas good projects yield gross return Γ with probability π and nothing with probability $1 - \pi$. Able entrepreneurs have only good projects, whereas incapable entrepreneurs have only bad projects. The fraction of able entrepreneurs is λ . The types of entrepreneurs are not known to investors and remain private information of the former at all times.

In the beginning of each period every entrepreneur meets one investor; the matching is uniform and random. Upon matching every entrepreneur presents a reference letter written by investor, who that entrepreneur worked for in the previous period. A letter states whether entrepreneur had honored the terms of his contract with that investor. The contracts available to entrepreneurs are debt contracts: an entrepreneur promises to pay the amount D (in the end of a period) to investor who makes him a unit loan (in the beginning of that period). The contracts are not directly enforceable: once a loan is made investor can neither verify the outcome of the project nor to seize entrepreneur's assets. Investor, however, can refuse a loan in the beginning or later, if the loan is not repaid, write a bad letter. Following Kreps and Wilson (1982), and others, let us say that entrepreneurs whose letters indicate that they have repaid their previous loan, have *good reputation* and those, whose letters say that they have not repaid the previous loan, have *bad reputation*. Assume that if an investor refuses to loan to an entrepreneur, then the latter ends up having bad reputation.

Finally, assume that the letters can be falsified: an entrepreneur who ends up with a bad letter can replace it with a good letter with probability ϕ and present that falsified letter to investor he meets in the next period (here ϕ is a measure of transparency in the banking sector). Investors cannot distinguish falsified letters and have to bear the risk that an entrepreneur who presents a good letter is a crook.

a) Let $P_{A|G}$ be the probability that conditional on having good reputation, a randomly met entrepreneur is able and let $P_{A|B}$ be the probability that conditional on having bad reputation, a randomly met entrepreneur is able. Compute $P_{A|G}$ and $P_{A|B}$. Which one is bigger? What are the probabilities $P_{I|G}$ and $P_{I|B}$? How do all four probabilities depend on ϕ ? Give an

intuitive explanation.

b) Consider a stationary equilibrium in which entrepreneurs with good reputation obtain the loans, entrepreneurs with bad reputation are refused the loans, and good entrepreneurs repay their loans whenever possible. Given that profile of equilibrium strategies, write down one-period expected returns of entrepreneurs in all four states $s \equiv (i, j)$, $i \in \{A, I\}$, $j \in \{G, B\}$, and the transition probabilities for reputation (for each of the two types). Let v_{ij} be the expected discounted utility of an entrepreneur in state s and let v be a 1×4 vector comprised of v_{ij} . Argue that the value function v is a solution to the system of stationary Bellman equations:

$$v' = \beta (q' + T v'),$$

where β is entrepreneur's discount factor, $0 \leq \beta < 1$, q is the 1×4 vector of one-period returns and T is the 4×4 matrix of transition probabilities. Use the Bellman equations to find the benefit of having good reputation, i.e. reputational capital:

$$v_{AG} - v_{AB},$$

of able entrepreneurs. How does it depend on patience, β ? Provide an intuitive explanation.

c) Write down conditions which guarantee that able entrepreneurs follow their equilibrium strategy of repaying the loans every time they have sufficient resources to do that. What is the maximum sustainable debt? How does it depend on ϕ ? Give intuition behind your findings.

d) What is the minimum amount of D , which will be preferred to investment in a riskless asset? State the parameter restrictions which guarantee that there is investment in projects at least for some D . Show that the only equilibrium in which there is investment in projects is the one that uses the profile of strategies introduced in part b). Give an intuitive explanation of why this is true.

e) Assume that anyone can freely enter the loan market. What is the equilibrium value of D under free entry? How is investment related to expected profitability, $\pi\Gamma - 1$, of good projects?

f) What do your findings imply for equilibrium investment? What variables would you include in an empirical test of the Tobin's q -theory? What is the role of transparency ϕ ? Explain.