Macroeconomics IV. 2004-2005. Homework 2.
Due: Thursday, April 7, 5:00pm.

1 The time–averaging problem

Suppose that consumption follows a random walk: $C_t = C_{t-1} + e_t$, where e is white noise. Suppose, however, that the data provide average consumption over two–period intervals; that is, one observes $(C_t+C_{t+1})/2$, $(C_{t+2}+C_{t+3})/2$, and so on.

(a) Find an expression for the change in measured consumption from one two–period interval to the next in terms of the e's.

(b) Is the change in measured consumption uncorrelated with the previous value of the change in measured consumption? In light of this, is measured consumption a random walk?

(c) Given your result in part (a), is the change in consumption from one two–period interval to the next necessarily uncorrelated with anything known as of the first of these two period intervals? Is it necessarily uncorrelated with anything known as of the two–period interval immediately preceding the first of the two–period intervals?

(d) Suppose that measured consumption for a two–period interval is not the average over the interval, but consumption in the second of two periods. That is, one observes C_{t+1}, C_{t+3}, and so on. In this case, is measured consumption a random walk?

(e) In light of the above results, what problems may emerge when testing the Hall (1978) version of the Permanent Income Hypothesis? What data are best suited for such tests?

2 Investment and the Real Business Cycle

Time is continuous and horizon is infinite. Consider a firm which owns its capital stock K_τ, employs one unit of inelastically supplied labor, and faces uncertainty about future productivity of the capital. Specifically, assume that the firm's production function is:

$$F(K_t) = A_t K_t^\alpha,$$

where A_t is time-t productivity and $\alpha \in (0,1)$. Further, assume that A_t can take two values, a and A, $0 < a < A$, where the transition between states a and A is governed by a Poisson process:

$$\Pr(A_\tau = A | A_t = A) = \Pr(A_\tau = a | A_t = a) = e^{-\kappa(\tau - t)}.$$

Here the arrival rate of the Poisson process κ, $\kappa > 0$, is a measure of frequency of productivity reversals. The firm seeks to maximize its expected discounted profits:

$$\pi_t = E_t \left[\int_t^\infty e^{-r(\tau-t)} \left(A_\tau K_\tau^\alpha - I_\tau \left[1 + \gamma \frac{I_\tau}{K_\tau} \right] \right) d\tau \right],$$

where E_t is the conditional on time-t productivity, A_t, mathematical expectation of future profits and $\gamma \frac{I}{K}$ is the per-unit adjustment costs of investment, $\gamma > 0$, and the initial capital stock is $K_t > 0$. Assume that physical depreciation of capital is zero, so that:

$$\dot{K}_\tau = I_\tau,$$

i.e. change in capital stock equals investment.

a) Describe how uncertainty affects firm's problem. Derive the first order conditions.

b) Derive the system of differential equations in K and q, which govern capital accumulation by the firm. Given $A_t = A$, depict the loci $\dot{K} = 0$ and $\dot{q} = 0$ on a phase diagram. Carefully explain why they look the way you draw them. Depict the loci $\dot{K} = 0$ and $\dot{q} = 0$ given $A_t = a$ on the same diagram.

c) Assuming that K_t is small enough, describe (in words) short-run capital and investment dynamics. Draw the appropriate picture. What is the effect of productivity shocks on capital accumulation and on investment? How investment dynamics changes if γ is set equal to zero? How does capital dynamics change when κ gradually approaches zero?

d) Redo part c) for the case of long-run dynamics. Comment on whether productivity shocks can lead to a cyclical dynamics of output. Discuss the link between your result and the notion of the real business cycle.