Macroeconomics IV. 2004-2005. Homework 1.
Due: Wednesday, March 23, 5:00pm.

Expectations, monetary policy, and the business cycle

Assume an economy, which is described by the two equations:

$$\begin{cases} y_t = \alpha(p_t - p_t^e) \\ y_t = m_t - p_t \end{cases}$$

where the first equation is aggregate supply equation and the second equation is aggregate demand equation. Here y_t, m_t, and p_t are (logarithms of) output, nominal money, and prices; p_t^e is time $t-1$ expectation of time t prices. Also, there is a monetary authority, who can use open-market operations to conduct monetary policy. Assume that nominal money follows

$$m_t = \rho m_{t-1} + \varepsilon_t,$$

where ρ is the policy choice of the authority. In particular, $\rho = 0$ means that the authority acts to keep nominal money stable; $\rho = 1$ means that the authority does not intervene and lets for unrestricted drifts in money. The shocks ε_t are identically and independently distributed with mean zero and variance σ^2. One can interpret these shocks as being the result of fluctuations in inside (i.e. generated by the banking system) money, which is beyond the grasp of policy makers. The policy objective of monetary authority is to minimize *unconditional* variance of output in every period t, i.e. to dampen the business cycle.

a) Assume that individuals who live in that economy are smart and form rational expectations,

$$p_t^e = E_{t-1} p_t.$$

What is the St. Louis equation[1] of the monetary authority? Solve for the

[1] A St. Louis equation is an equation, which relates current output to lagged output and current and lagged money:

$$y_t = \alpha + \sum_{i=1}^{k} \beta_i y_{t-i} + \sum_{i=0}^{k} \gamma_i m_{t-i}.$$

Such equations were routinely estimated by economists in the seventies. Especially, they were a popular research objective at the Federal Reserve Bank of St. Louis, which gave them that name.

unconditional variance of output as a function of model parameters. What is the optimal (i.e. variance minimizing) value of policy ρ?

b) Assume that there is a sudden shift in expectation formation, so that individuals become stupid and form static expectations,

$$p_t^e = p_{t-1}.$$

What is the St. Louis equation now? Is it different from that in part a)? Give an intuition behind your answer.

c) Solve for output as a function of current and past shocks to money. What is the unconditional variance of output? What value of ρ is optimal now? Does the optimal policy under static expectations allow to achieve a greater stability of output compared to that under rational expectations? Why or why not? Explain.

d) Assume now that a bit of common sense gets back into the heads of these people, so that they start forming adaptive expectations,

$$p_t^e - p_{t-1}^e = \lambda(p_{t-1} - p_{t-1}^e),$$

where λ, $0 \leq \lambda < 1$, is the fraction of surprise that passes onto expectations (assume that λ cannot be influenced by monetary authority). What is the St. Louis equation in that case? How is it different from above? Explain your answer.

e) Redo part c) for the case of adaptive expectations.

f) Suppose now that the authority can manipulate by public opinion about how much of innovation to prices is permanent (i.e. can influence λ). What is the message the authority will try to convince the public? What will be the corresponding optimal monetary policy? Give an intuitive explanation as a part of your answer. Would that policy yield more output stability that could be achieved under rational expectations?

Expectations, excess sensitivity and excess smoothness of consumption.

Assume an economy in which individuals are heterogeneous with respect to expectation formation. Specifically, assume that a fraction λ, $0 < \lambda < 1$, of a unit population forms static expectations and a fraction $1-\lambda$ forms rational expectations. The individuals (who live forever) face an uncertain income stream, $\{y_t\}_{t=0}^{\infty}$, where y_t follows a second-order autoregressive process,

$$y_t = (1+\rho)y_{t-1} - \rho y_{t-2} + \varepsilon_t, \tag{1}$$

where $\rho \in (0,1)$ and ε_t is *i.i.d.* with mean zero and variance σ^2. Each individual seeks to maximize expected (given her expectations — either static or rational) discounted utility of consumption; utility in a period is,

$$u(c) = c - \frac{a}{2}c^2, \tag{2}$$

where $a > 0$ and the period discount factor is β, $\beta \in (0,1)$. All individuals have access to a perfect asset market, which allows them to smooth consumption. Assume that interest rate is a constant r, $r > 0$, and that $\beta(1+r) = 1$.

a) Is the process for income (equation 1) stationary, trend-stationary, or difference-stationary? Prove your conjecture.

b) Formulate an individual optimization problem. Use a method of your choice to derive the Euler equation (one equation for each of the two groups of individuals).

c) Given the process for income, find consumption of both static expectations consumers and rational expectations consumers. What is the permanent income in both groups?

d) An economist called Mrs. Blabin thinks that everyone forms rational expectations and wants to test the permanent income hypothesis by running a regression:

$$\Delta c_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \epsilon_t, \tag{3}$$

(where ϵ_t is the error term. Assuming that Mrs. Blabin knows the process for income, compute estimates of the coefficients γ_1 and γ_2 in her regression.

Will they coincide with what Mrs. Blabin expects to find based on her assumption of the prevalence of rational expectations? What is the name you think Mrs. Blabin might want to associate with the observed evidence? How will Mrs. Blabin's task change if she does not know the process for income?

For the remaining part of the problem assume that λ is small.

e) Assume that another economist, Mr. Bali, shares Mrs. Blabin's presumption of the prevalence of rational expectations, but unlike Mrs. Blabin, wants to test the permanent income hypothesis by comparing the volatility (variance) of consumption with that predicted by theory (assume that Mr. Bali knows the process for income and the true value of σ). What is the volatility of consumption Mr. Bali expects to find? What is the volatility of consumption Mr. Bali will find? What is the name you think Mr. Bali might want to associate with his observation?