New Economic School
Macroeconomics 2
Optional problem set 6 answer key

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1. **True, False, Uncertain. Explain your answer.**

   (a) Uncertain. This is true if rational expectations are assumed as in NC theory only surprises matter, and past (observed) values of monetary growth are not surprises. However, there could be inertia in expectations (take adaptive expectations, for example), which may still leave a role for past shocks (as in our early discussion of AS-AD with $P_t^e = P_{t-1}$, or $\pi_t^e = \pi_{t-1}$).

   (b) False. It is the Neoclassical Macro (*not* rational expectations theory) that implies that anticipated monetary policy has no real effect. Unanticipated monetary policy does have real effect which can easily explain observed correlation of monetary growth and real magnitudes over the business cycle (compare this to question 2 in assignment 5).

   (c) False, false.

   The Neo-keynesian school of thought argues that monetary policy should be predictable because unpredictable policy component only adds to undesired economic volatility.

   Positive theory of Central Banking (not NK) asserts that people have higher inflationary expectations if CB cannot commit to a predictable policy and reserves discretion. This does not lead to higher than the natural rate of unemployment in equilibrium as it is the inflationary surprise factor that matters for unemployment and under rational expectations surprise is non-systematic (can only be generated by exogenous random process).

2. Stabilisation and commitment.

   Suppose the output is given by the Phillips curve equation rewritten in terms of y :

$$y = \alpha\left(\pi - \pi^e\right) + \varepsilon, \tag{1}$$

   where ε_t is an i.i.d. shock with variance σ^2 that occurs after private sector expectations are formed but before the Central Bank (CB) have to set monetary policy. The loss function that CB tries to minimise is

$$L = \pi^2 + \lambda\left(y_t - \bar{y}\right)^2,$$

   where $\bar{y} > 0$ is the CB's target output (note that the natural output level is zero as seen in (1).

   (a) FOC:

$$\begin{aligned} \pi + \lambda\alpha^2\pi &= \lambda\alpha^2\pi^e + \lambda\alpha\left(\bar{y} - \varepsilon\right) \Rightarrow \\ \pi &= \frac{\lambda\alpha^2}{1+\lambda\alpha^2}\pi^e + \frac{\lambda\alpha}{1+\lambda\alpha^2}\left(\bar{y} - \varepsilon\right) \end{aligned} \tag{2}$$

(b) Impose rational expectations, and find the equilibrium inflation for any realisation of ε_t.

$$\begin{aligned}
\pi^e &= E\pi = \frac{\lambda\alpha^2}{1+\lambda\alpha^2}\pi^e + \frac{\lambda\alpha}{1+\lambda\alpha^2}\bar{y} \\
\pi^e &= \bar{y}\alpha\lambda \\
\pi &= \frac{\lambda\alpha^2}{1+\lambda\alpha^2}\pi^e + \frac{\lambda\alpha}{1+\lambda\alpha^2}(\bar{y}-\varepsilon) \\
&= \lambda\alpha\left(\bar{y} - \frac{\varepsilon}{1+\alpha^2\lambda}\right)
\end{aligned}$$

(c) Compute equilibrium output for any given ε_t. Compute the expected value of the loss function.

$$\begin{aligned}
y_t &= \alpha(\pi_t - \pi^e) + \varepsilon_t \\
&= -\frac{\lambda\alpha^2\varepsilon_t}{1+\alpha^2\lambda} + \varepsilon_t = \frac{1-\lambda\alpha^2}{1+\alpha^2\lambda}\varepsilon_t
\end{aligned}$$

$$\begin{aligned}
L^D &= \pi^2 + \lambda(y_t - \bar{y})^2 \\
&= \left(\lambda\alpha\left(\bar{y} - \frac{\varepsilon_t}{1+\alpha^2\lambda}\right)\right)^2 + \lambda\left(\frac{1-\lambda\alpha^2}{1+\alpha^2\lambda}\varepsilon_t - \bar{y}\right)^2 \\
&= \bar{y}^2\lambda\left(1+\alpha^2\lambda\right) - 2y\lambda\frac{\varepsilon_t}{\alpha^2\lambda+1} + \frac{\left(\alpha^4\lambda^2 - \alpha^2\lambda + 1\right)\lambda\varepsilon_t^2}{\left(\alpha^2\lambda+1\right)^2} \\
EL^D &= \bar{y}^2\lambda\left(1+\alpha^2\lambda\right) + \frac{\left(\alpha^4\lambda^2 - \alpha^2\lambda + 1\right)\lambda}{\left(\alpha^2\lambda+1\right)^2}\sigma^2,
\end{aligned}$$

where $\sigma^2 = Var(\varepsilon)$.

(d) Compute the expected value of the loss function for the credible commitment to a rule $\pi \equiv 0$. Compare.

$$\begin{aligned}
y &= \alpha(\pi - \pi^e) + \varepsilon = \varepsilon \\
L^C &= \pi^2 + \lambda(y_t - \bar{y})^2 \\
&= 0 + \lambda(\varepsilon - \bar{y})^2 \\
EL^C &= \lambda\bar{y}^2 + \lambda\sigma^2
\end{aligned}$$

$$EL^D - EL^C = \bar{y}^2\alpha^2\lambda^2 - 3\frac{\lambda^2\alpha^2}{\left(\alpha^2\lambda+1\right)^2}\sigma^2 \gtrless 0$$

Depending on the volatility of disturbances ε, the commitment to fixed response may or may not be desirable