# New Economic School
# Macroeconomics 2

Problem set 3 selected answers

Instructor: Andrei Sarychev

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2. Recall problem 1 of the assignment 2. Using the AS-AD model of the lectures augmented with the expectations adjustment formula $P^e_{t+1} = P_t$, show the effects of improvements in monitoring on the position of the IS, LM, AD, and AS curves in the short, medium, and long term. Summarise the effects on output, interest rate, and the price level at all times.

   Aggregate demand equation:

$$y_t = m_t - p_t,$$

   The wage-setting equation is

$$w^X_t = w^X_{t+1} = p_t + \alpha y_t, \text{ where } X \in \{A, B\}$$

   The price setting equation in the economy is

$$p_t = \frac{w^A_t + w^B_t}{2}$$

   (a)

$$\begin{cases} y_0 = m_0 - p_0 = -p_0 \\ p_0 = w^A_0 = w^B_0 = p_0 + \alpha y_0 \end{cases} \Rightarrow$$

$$\begin{aligned} p_0 = w^A_0 = w^B_0 = y_0 &= 0, \\ \log\left(W^X/P\right) = w^X_0 - p_0 &= 0 \end{aligned}$$

   (b)

$$\begin{aligned} w^B_1 &= w^B_0 = 0 \\ w^A_1 &= p_1 + \alpha y_1 \\ p_1 &= \frac{w^A_1 + w^B_1}{2} = \frac{w^A_1}{2} \\ y_1 &= \bar{m} - p_1 \end{aligned}$$

Solving the system yields $p_1 = \frac{\alpha}{\alpha+1}\bar{m}, y_1 = \frac{\bar{m}}{\alpha+1}, w_1^A = \frac{2\alpha}{\alpha+1}\bar{m}$. Real wages:

$$w_1^A - p_1 = \frac{\alpha}{\alpha+1}\bar{m} > 0 > -\frac{\alpha}{\alpha+1}\bar{m} = w_1^B - p_1$$

(c)

$$\begin{aligned} w_2^A &= w_1^A = \frac{2\alpha}{\alpha+1}\bar{m} \\ w_2^B &= p_2 + \alpha y_2 \\ p_2 &= \frac{w_2^A + w_2^B}{2} = \frac{\alpha}{\alpha+1}\bar{m} + \frac{w_2^B}{2} \\ y_2 &= \bar{m} - p_2 \end{aligned}$$

$$\begin{aligned} w_2^B &= p_2 + \alpha y_2 \\ p_2 &= \frac{\alpha}{\alpha+1}\bar{m} + \frac{w_2^B}{2} \\ y_2 &= \bar{m} - p_2 \end{aligned}$$

Solving the system yields $p_2 = \frac{\alpha(\alpha+3)}{(\alpha+1)^2}\bar{m}, y_2 = \frac{(1-\alpha)}{(\alpha+1)^2}\bar{m}, w_2^B = \frac{4\alpha}{(\alpha+1)^2}\bar{m} > \frac{2\alpha}{\alpha+1}\bar{m}$ (the last inequality follows from $\alpha < 1$). The wages of metal workers are indeed higher than the miners'. The output is still above the steady state level. The real wages are not yet equal to 0.

(d)

$$\begin{aligned} w_t^L &= p_t + \alpha y_t \\ p_t &= \frac{w_{t-1}^L + w_t^L}{2} \\ y_t &= \bar{m} - p_t \end{aligned}$$

Solving yields a recursive relationship

$$w_t^L = \frac{2\bar{m}\alpha + w_{t-1}^L (1-\alpha)}{\alpha+1}$$

(e)

$$m_t - w_t^L = \bar{m} - \frac{2\bar{m}\alpha + w_{t-1}^L (1-\alpha)}{\alpha+1} = \frac{1-\alpha}{\alpha+1}\left(\bar{m} - w_{t-1}^L\right)$$

$\left\{m_t - w_t^L\right\}$ is the geometric series, and so is converging to zero. In the new steady state all real variables (logs of) are zeros as before, and the money shock is fully absorbed by nominal prices..

(f) Overlapping structure of contracts

3. Interest parity conditions.

Consider the price schedule for government bonds and foreign exchange in the United States and Russia. Both government bonds are one-year securities. The current exchange rate $e$ stands at \$0.2/1R (Russian currency is R for *rouble)*.

| Bond | Face Value | Price | Currency |
|---|---|---|---|
| US | 10,000 | 9,708.74 | US$ |
| Russia | 100,000 | 61,349.69 | *Rouble* |

a. Calculate the nominal interest rate on each of the bonds and the expected exchange rate next year consistent with Uncovered Interest Parity. Note whether this signifies an expected appreciation or depreciation of the *rouble*.
*Answer.* The nominal return on each of the bonds

$$i_{US} = \frac{FV_{US}}{P_{US}} - 1 = \frac{10,000}{9,708.74} - 1 = 0.03$$
$$i_{RU} = \frac{FV_{RU}}{P_{RU}} - 1 = \frac{100,000}{61,349.69} - 1 = 0.63$$

Also recall that the approximated uncovered interest parity is stated as $i_{US} - i_{RU} = \frac{e^e - e}{e}$
It easily follows that one can solve for $e^e$, the expected nominal exchange rate in one year: $e^e = e(i_{US} - i_{RU} + 1) = 0.2(0.03 - 0.63 + 1) = 0.08$
Since $e^e < e$ we have a depreciation of the rouble relative to the dollar.
b. Assume you exchange dollars for roubles and purchase the Russian bond, but one year from now it turns out that $e$ is actually \$0.05/1R. What is your actual nominal return compared to the return if you had just purchased the US bond? Are these differences in returns consistent with arbitrage?
*Answer:* The actual nominal return on the US bond remains 3%, but purchase of the Russian bond required a position in roubles, so the unexpected appreciation of the US dollar affects your return. Since you have to buy dollars when your bond matures and the price of dollars has unexpectedly risen, the nominal return from purchase of the Russian bond ex post is lower than the US bond. Mathematically, we can write this nominal return as follows:
$i_{RU} + \frac{e_{t+1} - e_t}{e_t} = 0.63 + \frac{0.05-0.2}{0.2} = -.12 < 0.03 = i_{US}$
Looking backwards one year from now, the nominal return falls short of that on the US bond. This does not mean, however, that no one should have bought Russian bonds given their information one year ago. It is also possible for there to be lower appreciation of the dollar than expected, which would turn your position in Russian currency into a fabulous opportunity.
The point of this question is that the type of arbitrage necessary for uncovered interest parity conditions to hold is arbitrage between *expected* returns. Although the use of the term "arbitrage" suggests free money, there is nothing riskless about uncovered positions in foreign currency.
We tend to think identical goods should have the same prices. If identical goods have different prices, it is possible to buy at the low price and sell at the high price and make lots of money. Arbitrage refers by actions by agents taking advantage of these kinds of price differentials. You should discern between two types of arbitrage. There is *expected* arbitrage, which is taking advantage of differences in *expected* return. This means you expect to make money, but will sometimes do worse and other times do

better, so you will make money on average. On the other hand there is *riskless* arbitrage, which means you will always make money regardless of the course of future events. The uncovered interest parity condition is an equilibrium condition equivalent to **no expected arbitrage opportunities** while the *covered* interest parity condition below is equivalent to **no riskless arbitrage opportunities**.

c. Assume that there exists a market for buying and selling foreign exchange one-year in the future, but fixing the price of the transaction today. Denote the forward price of one rouble in terms of dollars by $f$. In other words, you can enter into a contract today to sell one rouble for $f$ dollars one year in the future. Derive the following approximation to the Covered Interest Parity as stated below:

$$i_{US} = i_{RU} + \frac{(f-e)}{e}$$

*Answer.* The nominal gross return on the US bond is simply the nominal interest rate as before. On the other hand, instead of taking an uncovered position in foreign currency, consider a strategy of selling roubles forward in the amount you expect to have one year from today. One US dollar converts into $1/e$ rouble, which, invested in the Russian bond, will yield $\frac{1+i_{RU}}{e}$ roubles next year. Sell this amount forward today at a price of $f$ \$/R so the that nominal return on this investment strategy is $\frac{f(1+i_{RU})}{e}$. If nominal returns on each strategy are equal, then we have the following interest parity condition:

$$1 + i_{US} = \frac{f(1+i_{RU})}{e}$$

We can derive an approximation to covered interest parity:

$$i_{US} - i_{RU} = \frac{f-e}{e}$$

d. What is the forward price of 1 rouble consistent with Covered Interest Parity? Compare actual nominal returns between the two strategies if next year $e$ is actually 0.05 as above. Is Covered Interest Parity between the two 1-year bonds really riskless arbitrage?

*Answer.* Solving for $f$ in the interest parity condition above, it follows: $F = (1 + i_{US} - i_{RU})\, e = 0.2\,(1 + 0.03 - 0.63) = .08$.

Next years exchange rate has no effect on the nominal returns of either strategy, because we have covered our position in Russian currency by selling roubles forward. For one-year bonds, departures from covered interest parity are opportunities for riskless arbitrage.