Macroeconomics 2. Week6.
A practice question.
An open economy is described by the following equations

$$y_t = \bar{y} + \alpha(p_t - E_{t-1}p_t) \tag{AS}$$

$$y_t = -\beta r^* - \delta(e_t + p_t) + u_t \tag{IS}$$

$$m_t - p_t = \kappa y_t - \lambda r^* + v_t \tag{LM}$$

where $u_t, v_t$ haze zero means

Compute equilibrium output under the assumptions of fixed and flexible exchange rate regimes.
Fixed $\bar{e}$ : (AS)&(IS) solely determine $y$ and $p$, LM determines $m$

$$\bar{y} + \alpha\left(p_t - E_{t-1}p_t\right) = -\beta r^* - \delta\left(\bar{e} + p_t\right) + u_t \tag{AS=IS}$$

$$\bar{y} = -\beta r^* - \delta\left(\bar{e} + E_{t-1}p_t\right) \tag{$p^e$}$$

$$\left(\alpha + \delta\right)\left(p_t - E_{t-1}p_t\right) = u_t \tag{Equilibrium}$$

$$y_t = \bar{y} + \frac{\alpha}{\alpha + \delta}u_t \tag{1}$$

Flexible $e$ : (AS)&(LM) determine $y$ and $p$, (IS) determines $e$

$$\bar{m} - p_t = \kappa\left[\bar{y} + \alpha\left(p_t - E_{t-1}p_t\right)\right] - \lambda r^* + v_t \tag{AS+LM}$$

$$\bar{m} - E_{t-1}p_t = \kappa\bar{y} - \lambda r^* \tag{$p^e$}$$

$$\left(1 + \alpha\kappa\right)\left(p_t - E_{t-1}p_t\right) + v_t = 0 \tag{Equilibrium}$$

$$y_t = \bar{y} + \frac{\alpha}{1 + \alpha\kappa}v_t \tag{2}$$

A neo-Keynesian model

$$
\begin{aligned}
y_t &= \frac{\alpha}{2}(p_t - E_{t-1}p_t) + \frac{\alpha}{2}(p_t - E_{t-2}p_t) + u_t && (3) \\
&= \alpha(p_t - E_{t-1}p_t) + \frac{\alpha}{2}(E_{t-1}p_t - E_{t-2}p_t) + u_t && (4) \\
m_t - p_t &= \beta y_t + v_t && (5) \\
v_t &= \rho v_{t-1} + \varepsilon_t && (6)
\end{aligned}
$$

Taking expectations of the AD equation and subtracting:

$$(m_t - E_{t-1}m_t) - (p_t - E_{t-1}p_t) = \beta(y_t - E_{t-1}y_t) + (v_t - \rho v_{t-1})$$

where from (3), $E_{t-1}y_t = \frac{\alpha}{2}(E_{t-1}p_t - E_{t-2}p_t)$, so

$$(m_t - E_{t-1}m_t) - (p_t - E_{t-1}p_t) = \beta\alpha(p_t - E_{t-1}p_t) + \beta u_t + \varepsilon_t$$

Taking expectations at time $t-2$ of (5),

$$(m_t - E_{t-2}m_t) - (p_t - E_{t-2}p_t) = \beta(y_t - E_{t-2}y_t) + v_t,$$

where $E_{t-2}y_t = \frac{\alpha}{2}(E_{t-2}p_t - E_{t-2}E_{t-1}p_t) + \frac{\alpha}{2}(E_{t-2}p_t - E_{t-2}p_t) = \frac{\alpha}{2}(E_{t-2}p_t - E_{t-2}p_t) = 0$,so

$$(m_t - E_{t-2}m_t) - (p_t - E_{t-2}p_t) = \beta y_t + v_t$$

Hence,

$$
\begin{aligned}
(p_t - E_{t-1}p_t) &= \frac{1}{1+\alpha\beta}\left[(m_t - E_{t-1}m_t) - \beta u_t - \varepsilon_t\right] \\
(p_t - E_{t-2}p_t) &= (m_t - E_{t-2}m_t) - \beta y_t - v_t
\end{aligned}
$$

$$
\begin{aligned}
y_t = {} & \frac{\alpha}{2(1+\alpha\beta)}\left[(m_t - E_{t-1}m_t) - \beta u_t - \varepsilon_t\right] + \\
& \frac{\alpha}{2}\left[(m_t - E_{t-2}m_t) - \beta y_t - v_t\right] + u_t
\end{aligned}
$$

$$
\begin{aligned}
y_t = {} & \frac{\alpha}{(1+\alpha\beta)(2+\alpha\beta)}\left[(m_t - E_{t-1}m_t) - \beta u_t - \varepsilon_t\right] + \\
& \frac{\alpha}{(2+\alpha\beta)}\left[(m_t - E_{t-2}m_t) - v_t\right] + \frac{2u_t}{2+\alpha\beta}
\end{aligned}
$$