New Economic School
Game Theory II – Makeup Exam

May 23, 2005

This is an open-book exam. You have three hours to answer the three questions below. Questions 1 and 2 count for 30% of the exam grade each while question 3 counts for the other 40%. Good luck!

1. Bayesian games. There are two agents, firm F and consumer C. The firm may sell a widget to the consumer at (fixed) price \$1. The widget may be either of high or of low quality. Firms costs of producing high quality widget is \$1, those of producing low quality widget is \$0.50. High quality widget delivers utility \$1 to the consumer (in monetary terms) while low quality widget delivers utility zero to her.

   The firm moves first and chooses the quality level. Once the consumer buys the widget she learns its quality and if it is low she may sue the firm. If she wins the litigation the firm must fully refund (i.e., give back the price) and also cover the litigation costs \$0.50. If the consumer loses the litigation the consumer gets no refund and also has to cover the litigation costs (same \$0.50).

   The litigation is won by the party whose lawyer is better skilled (if equally, the court flips a fair coin). Skills θ_F of firm's lawyer and skills θ_C of consumer's lawyer are independent random variable uniformly distributed on $[0, 1]$. Each party knows the skills of its lawyer but not those of the other party's lawyer.

   The firm cares about profits, the consumer cares about the utility of the widget minus the costs. Both are risk neutral.

   (a) Does there exist a strong sequential (aka perfect Nash-Bayes) equilibrium in which the firm produces low quality widget regardless on θ_F?
   (b) Does there exist a strong sequential equilibrium in which the firm may produce high quality widget at some level of θ_F?

2. Bilateral bargaining. Ann has tea in quantity 1 and Betty has sugar in quantity 1. Ann's utility of tea and sugar is $u_A(t_A, s_A) = \sqrt{t_A s_A}$ while Betty's utility of tea and sugar is $u_B(t_B, s_B) = \sqrt[4]{t_B s_B}$. They have to agree to trade some tea for some sugar.

   (a) Formulate the bargaining problem and draw it in utility's axes. What is the disagreement point?
   (b) Find the λ-utilitarian, λ-egalitarian, Nash and Kalai-Smorodinsky solutions.

3. Parliament games and social choice. Consider a parliament with four members: Boris, Dmitry, Genady and Vladimir. A bill is passed either if it is approved by at least three members or if it is approved by Boris and another member.

   (a) Is the game superadditive? Supermodular?
   (b) Find the core of the game.
   (c) Find the Shapley value of the game.
   (d) Find the nucleolus of the game.
   (e) Think of this parliament through the prism of social choice theory. There are two outcomes (pass and fail) and each of the four members has strict preferences over these outcomes. Each member approves the bill if he prefers it passed and disapproves if he prefers it failed (ignore all strategic considerations). The bill passes or fails according to the rules above, which generates social preferences from the set of individual preferences, i.e., defines a preference aggregator. Is this preference aggregator Paretian? Does it satisfy independence of irrelevant alternatives? State Arrow's impossibility theorem. Is this aggregator dictatorial?