

New Economic School
Game Theory
Problem Set #7

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Due Friday, March 24, 2006, at 20.00

1. (From 2005 final exam) An old lady and her physician are bargaining about the costs of her consultation, which can be anything between 0 and 1. If they agree, she enjoys the benefit of 1 less the amount p she pays; the physician though feels bad about charging the old lady so he enjoys only $p - \alpha \frac{p^2}{2}$ if he is paid p , where $\alpha \in [0, 1]$ measures his conscience (the physician incurs no costs to consult the lady). If they do not agree, the old lady does not come for the consultation and both get zero.
 - (a) Represent bargaining problem (X, d) graphically.
 - (b) Find λ -egalitarian solution (your answer would depend on λ and α).
 - (c) Find λ -utilitarian solution.
 - (d) Find Nash bargaining solution.
 - (e) Find Kalai-Smorodinsky solution. Does there exist α such that it coincides with the Nash solution?
 - (f) For this question assume that $\alpha = 0$. For a fixed integer $T > 0$ consider the following bilateral bargaining game: the lady moves first and offers p_1 ; the physician can either accept, in which case the game ends, or reject, in which case he can make an alternative offer p_2 . The lady may either accept the offer, in which case the game ends, or reject, in which case she can make another offer p_3 and so on. However, the game is finite: if offer p_{2T} is not accepted, the parties earn zero each and do not interact again. The parties face the same discount factor $\delta \in (0, 1)$. Find subgame perfect equilibrium payoff pair and show that it is unique. Find payoff limits as $T \rightarrow \infty$.

2. Two firms live forever (discounting with factor $\delta < 1$) and in each period compete à la Cournot. Their marginal costs are zero and the industry demand in period $t = 0, 1, \dots$ is $p^t = A^t - q_1^t - q_2^t$. Here A is the demand parameter which can take values $A_l = 1$ and $A_h = 2$. The firms observe demand A^t prior to choosing q_i^t . The probability that the value of A remains constant from a period to the next is $\pi(A_h^{t+1}|A_h^t) = \pi(A_l^{t+1}|A_l^t) = \frac{2}{3}$. Initially $A^0 = 2$.
- What is the maximum feasible (expected) payoff? Is it sustainable as a Nash equilibrium in an infinitely repeated game with large enough discount factor δ ?
 - Find all Markov perfect equilibria in a game with two states (corresponding to high and low demand). Take the limit as $\delta \rightarrow 1$. Compare firm's payoffs to (a).

Assume now that there is federal antitrust service that may intervene at any period (prior to firms choosing their quantities). If it does, it relaxes import quotas which means that there will be inflow of import substitute in the quantity of 1. In period t the intervention happens with probability $\varphi_t = \min\{\alpha \cdot p_{t-1}, 1\}$, where p_{t-1} is the market price in the previous date (i.e., the higher the price, the more likely the antitrust service to intervene) and is independent on the demand evolution or on past interventions.

- For a given α find a symmetric Markov perfect equilibrium that delivers maximum profits to the firms and compare with (b). Are there other Markov perfect equilibria? In what follows, assume that whenever the game is played, the firms play symmetric Markov strategies that yield them maximum payoff.
- Assume you are the antitrust authority. All you care about is consumer surplus and your own intervention costs. Prior to the game described above you have to setup α , which costs you $c(\alpha) = \frac{C\alpha^2}{2}$ (where C is the costs of enforcing the antitrust law). What level of α will you choose?
- Assume now that you are the social planner. You care about the consumer surplus and profits of the two firms (but not about profits of foreign firms). Your antitrust agency wants to keep the price low and open the market for foreign goods, but you have to balance it against protecting the profits of domestic firms. What level of C will you pick?