New Economic School
Game Theory
Problem Set #6

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Due Tuesday, March 14, 2006, at 21.00

1. (From 2005 final exam) The family consists of the father, the mother and the daughter; they want to spend a Saturday together. There are three options: go to a soccer game, go to Lenkom and go to the zoo, and each family member independently decides where to go. The father prefers soccer, the mother prefers Lenkom and the daughter prefers the zoo, but they also want to spend time together. Going to one's preferred entertainment delivers payoff $3 + 2k$ and going to an entertainment other than the preferred one delivers payoff $2k$, where k is the number of other family members going to the same entertainment.

   (a) Write down the game in normal form (one player chooses rows, one chooses columns and one chooses matrices).

   (b) Find all pure strategy equilibria of the game.

   (c) Find a symmetric mixed strategy equilibrium of the game or show than none exists.

   (d) Does there exist a correlated equilibrium that delivers payoffs $(5, 5, 5)$ to the players? If so, construct such an equilibrium; if not, show that it does not exist.

   (e) Does there exist a correlated equilibrium that delivers payoffs $(7, 6, 3)$ to the players? If so, construct such an equilibrium; if not, show that it does not exist.

   (f) Assume the game is played four times (with no discounting). Does there exist a subgame perfect equilibrium that involves the father going to the zoo, the mother going to the soccer game and the daughter going to Lenkom in the first period? If so, (fully) describe player's strategies. If not, show why.

2. (From 2005 final exam) Consider the following game in normal form:

	b_1	b_2	b_3
a_1	$2,3$	$3,1$	$2,2$
a_2	$1,0$	$3,5$	$3,2$

(a) What is the set of rationalizable strategies for each player? For each rationalizable strategy r of each player give a strategy q of his opponent to which r is a best response (if such strategy q exists).

(b) What are the pure strategy Nash equilibria of the game?

(c) Draw the set of payoffs attainable in a correlated equilibrium.

(d) The game drawn above is played infinitely many times and players discount future payoffs exponentially with discount factor δ. Draw the set of subgame perfect equilibrium payoffs (when $\delta \to 1$).

3. There are two firms that compete a là Cournot. Their marginal costs are zero and the demand is $q = \max\{1 - p, 0\}$. Each period $t = 0, 1, \ldots,$ they set quantities q_1^t and q_2^t and the market clears at $p^t = 1 - q_1^t - q_2^t$. At the end of each period both firms observe p^t. The discount factor between periods is $\delta \in (0, 1)$.

(a) On the payoff plain draw the set of payoffs attainable to the firms in a subgame perfect equilibrium as $\delta \to 1$.

(b) For Nash trigger (infinite punishment) strategies calculate the maximum attainable payoff in a subgame perfect equilibrium for a given δ.

(c) Consider the following (symmetric) pair of strategies:

- produce q^* in the first period and in each subsequent period until someone defected or if both defected in the last period;
- if you were the last to defect and you you did so in (at least) one of the last T periods, produce q_d; if your opponent were the last to defect and your opponent did so in (at least) one of the last T periods, produce q_p;
- if the last defection occurred more than T periods ago, produce q^*.

Find conditions on T, q^*, q_d and q_f that makes this pair of strategies a subgame perfect equilibrium for a given δ. For each T find maximum (equal) payoffs that can be supported by such pair of strategies and compare with your answer in (b). What is the optimal length of punishment T?