**Liquidation threat and stochastic investor control**
based on Bolton and Scharfstein (1996)

## Main idea:

- **Optimal contract involves trade-off between deterring strategic default (ex-ante efficiency) and inefficient liquidation following liquidity default (ex-post efficiency)**
  - **Giving control to investors in case of default deters entrepreneur from strategic default but may provoke too much inefficient liquidation**
  - **Hence, giving investors control with probability strictly below 1 can be optimal**
- **The resulting optimal contract is a debt-like contract:**
  - **Entrepreneur keeps control in case of no default**
  - **In case of default, control goes to investors with some probability and then liquidation happens**

# The Model

- **At** $\mathbf{t = 0}$, an entrepreneur raises $I$ (investment outlay), from investor(s), to finance two-period investment project ($NPV > 0$).
  $I$ is spent on buying a physical asset needed for production
  - Competitive capital markets
- **At** $\mathbf{t = 1}$, assets generate random first-period return and continuation/liquidation decision has to be taken
  - Cash flow $X_1 \in [0, X^S]$
    - with $p := \Pr[X_1 = X^S]$
  - In case of liquidation:
    - Liquidation value of the assets $L$
    - Assumption: $0 < L < I$
    - Note: we will allow for stochastic liquidation (with prob. $\beta$) but not for partial liquidation (Bolton and Scharfstein show it is suboptimal in their framework)
- **At** $\mathbf{t = 2}$, if continuation decision was taken, assets generate second period return
  - Cash flow $X_2$
    - deterministic
    - $X_2 > L$
  - Liquidation value of remaining assets: $L_2 = 0$

## First Best

Given project has positive NPV, i.e.,

$$pX^S + X_2 > I$$

$\longrightarrow$ it should be undertaken

Given liquidation at $t = 1$ is inefficient, relative to keeping assets inside the firm, i.e.,

$$X_2 > L$$

$\longrightarrow$ no liquidation, i.e., $\beta = 0$ at $t = 1$

**Assumption**: Cash flows $X_t$ are observable but *not verifiable*

$\longrightarrow$ Symmetric information, but contracts can only be made contingent upon

- – repayments by entrepreneur
  – and firm's physical assets (control over continuation/liquidation)

The contract specifies the probability the investor can liquidate, given repayment $R$: $\beta(R)$

## Full Commitment Contract

Given $L_2 = 0$, there is no liquidation threat at $t = 2$, and investors cannot induce manager to repay anything at $t = 2$

In contrast, at $t = 1$ investor can credible threaten to liquidate part of the firm

Revelation principle: we can restrict our attention to contracts with two points: $(R^S,\, \beta^S)$ and $(R^F,\, \beta^F)$

Thus, the contracting problem is

$$\max_{R^S, \beta^S, R^F, \beta^F} p[X^S - R^S + (1-\beta^S)X_2] + (1-p)[-R^F + (1-\beta^F)X_2]$$

$$X^S - R^S + (1-\beta^S)X_2 \geq X^S - R^F + (1-\beta^F)X_2 \qquad (IC)$$

$$p[R^S + \beta^S L] + (1-p)[R^F + \beta^F L] \geq I \qquad (IR)$$

$$R^F \leq 0 \text{ and } R^S \leq X^S \qquad (LL)$$

$$0 \leq \beta^S \leq 1 \text{ and } 0 \leq \beta^F \leq 1$$

**Remark**: In principle, there is a second IC: Paying $R^F$ when $X = X^F = 0$ must be preferred to paying $R^S$. However, since $R^S > 0$, it is never feasible to pay $R^S$ when $X = 0$.

**Solution:**

- $\beta^{S*} = 0$: maximizes surplus from contract and entrepreneur's incentive to repay $R^S$

Suppose $R^S < X^S$ and $\beta^S > 0$

Reducing $\beta^S$ by $\varepsilon$ and increasing $R^S$ by $\varepsilon L$ does not affect (IR) and relaxes (IC), while entrepreneur gains $p\varepsilon(X_2 - L)$.

In principle we should make sure that $R^S$ does not hit $X^S$, but let's assume for the moment $R^S < X^S$ in the optimum.

- $R^{F*} = 0$: also maximizes surplus from contract and entrepreneur's incentives

Suppose $R^F < 0$: Increasing $R^F$ by $\varepsilon$ and reducing $\beta^F$ by $\varepsilon/L$ does not affect (IR) and relaxes (IC), while entrepreneur gains $(1-p)\varepsilon((X_2/L) - 1)$.

In principle we should make sure that $\beta^F$ does not hit $0$, but let's assume for the moment $\beta^F > 0$ in the optimum.

- $R^{S*} = \beta^F X_2$: IC constraint binds; otherwise $R^S$ could be raised by $\varepsilon/p$ and $\beta^F$ reduced by $\varepsilon/((1-p)L)$, leaving (IR) unaffected and increasing the entrepreneur's gain

If $\beta^F X_2$ is larger than $X^S$, LL binds and $R^S = X^S$ Then optimal contract requires $\beta^S > 0$. Let's assume for the moment $\beta^F X_2 < X^S$ in the optimum.

- $\beta^{F*}$ is such that IR is binding. Otherwise, $R^S$ could be reduced, relaxing (IC) while increasing the entrepreneur's gain.

$\rightarrow$ Provided that $pX_2 + (1-p)L \geq I$

and that $X^S$ is sufficiently large, i.e., $X^S \geq \dfrac{I}{pX_2 + (1-p)L} X_2$,

the **optimal full commitment contract** is given by

$$R^{F*} = 0 \qquad \text{and} \qquad \beta^{F*} = \frac{I}{pX_2 + (1-p)L}$$

$$R^{S*} = \beta^F X_2 = \frac{I}{pX_2 + (1-p)L} X_2 \quad \text{and} \qquad \beta^{S*} = 0$$

Indeed $R^{S*} < X^S$, $\beta^{F*} > 0$, so our above reasoning is valid

**Investors' (gross) payoff is** $\beta^{F*}[pX_2 + (1-p)L] = I$.

Thus, in the optimal full commitment contract, inefficient (partial) liquidation always takes place at $t = 1$ if $X_1 = 0$ and never if $X_1 = X^S$.

(Implicit assumption: $X^S$ is sufficiently large)

## Comments

- Since (IR) binds, the optimal contract minimizes the aggregate expected loss from liquidation s.t. the constraints.
- There is inefficient liquidation when $X = 0$
- Inefficient liquidation is necessary to prevent strategic defaults.
- Optimal contract may be roughly interpreted as debt contract
  - entrepreneur borrows $I$ and promises to repay $R^S$
  - if entrepreneur defaults, creditor seizes the assets with some probability
  - Though, **standard** debt contract gives creditor right to seize the firm's assets **with probability one** in default. Setting $\beta^F = 1$ is, however, suboptimal as it leads to excessive (inefficient) liquidation.
- The full-commitment optimal contract is **not renegotiation-proof**

## Renegotiation

Liquidation is always inefficient ($X_2 > L$)

If liquidity default ($X = 0$), entrepreneur cannot offer anything in exchange for less liquidation

If strategic default ($X = X^S$), there is room for renegotiation when investor(s) get the right to liquidate. Entrepreneur can offer investor(s) more than liquidation proceeds $L$.

If no default and investors get the right to liquidate (with prob. $\beta^S$) there may also be a room for renegotiation. But Bolton and Scharfstein ignore it for some reason (they assume that liquidation occurs). We will not ignore it (though it does not affect the solution).

Let us not be specific about renegotiation rules for the moment. Just assume entrepreneur obtains:

- $S$ when investor(s) get the right to liquidate after a strategic default
- $S_N$ when investor(s) get the right to liquidate after no default

To find **optimal renegotiation-proof (after strategic default)** contract we need to solve:

$$\max_{R^S,\beta^S,R^F,\beta^F} p[X^S - R^S + \beta^S S_N + (1-\beta^S)X_2] + (1-p)[-R^F + (1-\beta^F)X_2]$$

$$X^S - R^S + \beta^S S_N + (1-\beta^S)X_2 \geq X^S - R^F + \beta^F S + (1-\beta^F)X_2 \qquad (IC')$$

$$p[R^S + \beta^S(X_2 - S_N)] + (1-p)[R^F + \beta^F L] \geq I \qquad (IR')$$

$$R^F \leq 0 \text{ and } R^S \leq X^S \qquad (LL)$$

$$0 \leq \beta^S \leq 1 \text{ and } 0 \leq \beta^F \leq 1$$

Following similar reasoning, the **optimal renegotiation-proof (after strategic default) contract** is given by

$$R^{F\prime*} = 0 \qquad \text{and} \qquad \beta^{F\prime*} = \frac{I}{p(X_2 - S) + (1-p)L^L}$$

$$R^{S\prime*} = \beta^{F\prime*}(X_2 - S) = \frac{I(X_2 - S)}{p(X_2 - S) + (1-p)L} X_2 \quad \text{and} \qquad \beta^{S\prime*} = 0$$

**Investors' (gross) payoff is** $\beta^{F\prime*}[p(X_2 - S) + (1-p)L] = I$.

Note: in principle $\beta^{S\prime*}$ can be $> 0$ in the optimum as there's no liquidation in case of no default. But setting it 0 does not change anything. So let's stick to Bolton-Scharfstein (under their assumption of possibility of liquidation after no default, setting $\beta^{S\prime*} = 0$ is strictly optimal).

Possibility of renegotiation leads to

$R^{S\prime*} < R^{S*}$: To prevent strategic default, investor can extract less from entrepreneur in good state.

$\beta^{F\prime*} > \beta^{F*}$: To recoup $I$ (in expected terms) investor needs higher returns following a liquidity default to compensate for lower return $R^{S\prime}$.

$\rightarrow$ larger agency costs of outside financing, i.e., more inefficient liquidation

## Comparative Statics:

- $\beta^{F\prime}$ (and inefficiency) increase with $S$ (entrepreneur's bargaining power):
- Inefficiency decreases with $L$ for two reasons:
  - liquidation value itself rises
  - probability of inefficent liquidation, $\beta^{F\prime}$, falls

**Allocation of control and protection of entrepreneur's non-contractible investment**
Tirole, ch. 10.2.4.

**Main idea:**

- **So far investor control has never decreased the pledgeable income**
  - **it always increased the investors' (gross) return, thus, making financing more likely to be feasible (though it could result in ex-post inefficiency)**
- **If investor control adversely affects entrepreneur's specific investment that create value for investors, investor control may decrease pledgeable income (hence, make financing less likely)**

## The model

- $A = 0$, for simplicity
- Investment needed: $I > 0$
- There is a status quo project that yields $X \in \left\{0, X^S\right\}$, $\Pr\{X = X^S\} = p$, no private benefit.
- At $t = 0$: Financing
- At $t = 1$: Specific investment (initiative) stage
  - The entrepreneur can either:
    - incur cost $c$ and find alternatives to the status quo
    - or does not incur $c$ – then only status quo is feasible
  - If he incurs $c$ he finds two alternatives:
    - A: increases $p$ by $\tau_A > 0$, yields private benefit $b_A > 0$
    - B: increases $p$ by $\tau_B > \tau_A$, yields private benefit $b_B < b_A,\ b_B > 0$
- At $t = 2$: A, B or "status-quo" is implemented. Returns and private benefits are realized.

- **Assumption:** no financing if no initiative: $pX^S < I$
- **Assumption:** when entrepreneur finds the alternatives, they become known to all parties, i.e. no asymmetry of info.
- **Assumption:** choice of an alternative is not contractible ex-ante
  - Hence, the contract can only specify repayment $R^S$ and the allocation of control at $t = 1$.
- **Assumption:** B is ex-post efficient, and initiative is desirable: $\tau_B X^S + b_B > \tau_A X^S + b_A > c$
  - Hence, investor control is ex-post (*after* $c$ is incurred) efficient.

## Investor control

- Investor chooses B
- No renegotiation occurs (B is ex-post efficient)
- Entrepreneur demonstrates initiative iff

$$\tau_B(X^S - R^S) + b_B \geq c$$

or

$$R^S \leq X^S + \frac{b_B - c}{\tau_B} \qquad (\mathrm{IC}_B)$$

- Investor provides funds iff

$$(p + \tau_B)R^S \geq I$$

- Hence, financing occurs iff

$$(p + \tau_B)\left(X^S + \frac{b_B - c}{\tau_B}\right) \geq I \qquad (*)$$

## Entrepreneur control

- Assume $R^S$ is such that entrepreneur wants to do B
- For this we need that $\tau_B(X^S - R^S) + b_B > \tau_A(X^S - R^S) + b_A$, i.e.

$$R^S \leq X^S - \frac{b_A - b_B}{\tau_B - \tau_A} \qquad (\mathrm{IC}_{B \succ A})$$

- Then no renegotiation
- Investor provides funds iff

$$(p + \tau_B)R^S \geq I$$

- Given that ($\mathrm{IC}_B$) must hold too, financing occurs iff

$$(p + \tau_B)\min\left\{X^S + \frac{b_B - c}{\tau_B},\, X^S - \frac{b_A - b_B}{\tau_B - \tau_A}\right\} \geq I \qquad (**)$$

(**) is stronger than (*)

- Assume $R^S$ is such that entrepreneur wants to do A, i.e. (IC$_{B \succ A}$) does not hold.

- Then there is always renegotiation and ultimately B is implemented.

- Assume entrepreneur has full bargaining power. Entrepreneur will sell control to investor in exchange for lower $R$, $R'$ such that:

$$(p + \tau_B)R' = (p + \tau_A)R^S$$

- Entrepreneur's payoff is $(p + \tau_B)(X^S - R') + b_B - c = (p + \tau_B)X^S - (p + \tau_A)R^S + b_B - c$

- Entrepreneur demonstrates initiative iff

$$\tau_B(X^S - \frac{\tau_A}{\tau_B}R^S) + b_B \geq c$$

or

$$R^S \leq \frac{\tau_B}{\tau_A}\left(X^S + \frac{b_B - c}{\tau_B}\right) \qquad (\text{IC}_A)$$

- Investor provides funds iff

$$(p + \tau_B)R' = (p + \tau_A)R^S \geq I$$

- Hence, financing occurs iff

$$\left(p\frac{\tau_B}{\tau_A} + \tau_B\right)\left(X^S + \frac{b_B - c}{\tau_B}\right) \geq I \qquad (***)$$

- Since, by assumption, $\tau_B X^S + b_B > c$, (***) is *weaker* than (*)
- Hence financing is more likely under entrpepreneur control in this case!
- **Note:** For entrepreneur conrol to be better, it must be that there exists $R^S$ such that:
  - (IC$_{B \succ A}$) does not hold and
  - (IC$_A$) holds at the same time

  this implies:

$$X^S - \frac{b_A - b_B}{\tau_B - \tau_A} < \frac{\tau_B}{\tau_A}\left(X^S + \frac{b_B - c}{\tau_B}\right)$$