New Economic School
Contract Theory
Problem Set 1

Andrei Bremzen
T/A: Konstantin Bayandin, Andrei Savochkin

Due Monday, November 12, 2007, 9.00

1. A worker can choose either high or low level of effort. The cost of high level is 1, the low level does not cost anything. If the agent chooses the high (low) level, principal gets π with probability p_H (p_L) and zero otherwise. The agent's reservation utility is zero. The principal can offer the agent a contract contingent on the principal's payoff. The agent's utility of monetary payments is

$$u(x) = \begin{cases} x, & x > 0; \\ Rx, & x < 0, \end{cases}$$

where $R > 1$. For all parameter values:

   (a) Find the first best level of effort.

   (b) Assume that the effort is not observable. Find the optimal contract. For which parameter values does it implement first best?

2. Consider a model where there are two risk-neutral principals (P1 and P2) and two risk-neutral agents (A1 and A2). Each principal's utility depends on the effort of corresponding agent but there is also an externality:

$$x_1 = a_1 + \delta_1 a_2, \; x_2 = a_2 + \delta_2 a_1,$$

where $\delta_{1,2} \in (0, 1)$. Principal Pi maximizes $x_i - w_i$, where w_i is the total wage paid out by Pi. The cost of effort of agent Aj is $c_j(a_j) = a_j^2/2$. The reservation utility of each agent is 0.

   (a) Describe the first best.

   (b) Consider the following game. First, principals non-cooperatively choose the wage schedules $w_i(\cdot)$ (here $w_i(x_i)$ is the wage that Pi pays Ai if x_i is observed). Then agents non-cooperatively choose their effort levels. Then x_1 and x_2 are observed, and the wages are paid. Find the subgame perfect Nash equilibrium(a). Compare to the first best.

   (c) Suppose that principals can pay wage to each agent, instead of choosing $w_i(\cdot)$, they now set $w_{i1}(\cdot)$ and $w_{i2}(\cdot)$ where $w_{ij}(x_i)$ is the wage paid by Pi to agent Aj if x_i is observed. Find the subgame perfect Nash equilibrium(a). Compare to the first best.

3. If the agent exerts effort a, the probability that the outcome (which accrues to the principal) is 1 equals a (with the remaining probability $1-a$ the outcome is 0). Both the principal and the agent are risk neutral, but the agent, whose reservation utility is zero, is liquidity constrained, which means that the principal can only offer nonnegative wage $w(x) \geq 0, \quad x = 0, 1$. Exerting effort costs the agent $C(a)$, where $a \in [0,1]$, $C'(a) > 0$, $C''(a) > 0$, $C'(0) = 0$, $C'(1) = \infty$.

   (a) Find first best level of effort a^{FB}.

   (b) Find the optimal (from the principal's point of view) contract and compare the agent's effort that it induces to a^{FB}.

4. (From 2004 final exam) There is a risk neutral principal and a risk averse agent. The agent may engage in three unobservable activities: a, a_1 and a_2 at costs $C(a, a_1, a_2) = \frac{(a+a_1+a_2)^2}{2}$. Activity a is productive for the principal: it results in verifiable output $x = a + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$ (note that a may be positive or negative!). Activities a_1 and a_2 are not productive for the principal, but the agent enjoys private benefits $\nu_1(a_1) = \frac{4}{3}\sqrt{a_1}$ and $\nu_2(a_2) = \frac{4}{3}\sqrt[4]{a_2^3}$. Agent's utility as a function of monetary compensation w and effort spent on each of three activities is $u(w, a, a_1, a_2) = -e^{-(w - C(a,a_1,a_2) + \nu_1(a_1) + \nu_2(a_2))}$ and his reservation utility is -1. The principal designs a linear contract in the form $w(x) = \alpha x + \beta$; in addition she can ban either of the two side activities (or both).

   (a) Find first best level of a, a_1 and a_2. Which activities would the principal ban if a, a_1 and a_2 were observable?

   (b) Find agent's choice of a, a_1 and a_2 as functions of α.

   (c) For a fixed $\alpha \in [0,1]$, which activities would the principal want to ban?

   (d) Assume $\sigma^2 = 99$. Find optimal α and activities ban list.

   (e) Assume $\sigma^2 = 2$. Find optimal α and activities ban list. For simplicity assume that the principal bears additional prohibitively high costs if he allows both private activities.

   (f) Qualitatively, how will optimal α and activities ban list change as σ^2 grows from zero to infinity?

5. As in class, there is a risk neutral principal and a risk averse agent. The agent exerts effort a that results in output $x = a + \varepsilon$, where $\varepsilon \sim N(0,1)$. The principal observes x but not a. The agent's utility is $u(w,a) = -\frac{1}{2}e^{-2(w-c(a))}$ if he works for the principal and $-\frac{1}{2}e^2$ if he does not; his costs of exerting effort $a > 0$ are $c(a) = \frac{ka^2}{2} + \frac{k+1}{2k(2k+1)}$ for some parameter $k \in [1,2]$.

   (a) For a given k, find the first best level of effort. Calculate the principal's profit as a function of k and find $k \in [1,2]$ that gives maximum first best profit to the principal.

   (b) Solve for the second best linear contract $w(x,k) = \alpha(k)x + \beta(k)$. Calculate the second best principal's profit as a function of k and maximize it over $k \in [1,2]$. Compare your answer to the first best case and provide intuition.